Similarity estimation for computerize footwear fit

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Abstract

As consumers are becoming increasingly selective of what they wear on their feet, manufacturing encountered the problem of developing right footwear. It is widely accepted that the three-dimensional model of foot can help in good shoe fitting.
Footwear fitter have been using manual measurement for a long time, but the combination of 3D scanning systems with mathematical technique makes possible the development of systems, which can help in the selection of good footwear for a given customer. In this paper, we proposed new approach for finding footwear fit within the shoe last data base.
Our new approach is based on the efficient algorithm for cutting 3D triangle mesh to several sections toward heel and toe. Then the area of each contour is calculated and compared with area of equal section in shoe last data base for finding footwear fit.
The first step is to fill holes in triangle mesh; after solving this post-process problem, our method is applied for finding footwear fit within shoe last data base.

Keywords: *shoe last, footwear, mesh, hole.*

1. INTRODUCTION

Very few standards exist for fitting products to people. Footwear fit is a noteworthy example for consumer considerations when purchasing shoes. As a result, the footwear manufacturing industry, in order to achieve commercial success is challenged to produce the differing customer preferences through product variety.
Loose shoes (even thought function may be impaired) are not as uncomfortable as when the shoes are tight. Properly constructed footwear may provide the right pressure and force at the different locations on the foot surface, and this may result in improved comfort, fit and foot health.
Unlike any other consumer product, personalized footwear or the matching of footwear to feet is not easy if delivery of discomfort is predominantly caused by localized pressure induced by a shoe that has a design unsuitable for that particular shape of foot [1].
The design of new shoes starts with the design of the new shoe last. The shoe last is a wooden or metal model of human foot on which shoes are shaped.
Nowadays, with development of 3D acquisition devices, automatic process, producing custom-tailored footwear is reasonable, if the custom last can be automatically produced based on consumer's foot shape.
A new approach to computerize footwear fit proposed in this paper. Due to both the object complexity and scanning process, some areas of the object outer surface may never be accessible, thus obtained data typically contains missing pieces and holes. However, this deficiency is not acceptable, where the geometric models are using in design process in manufacturing. Thus certain repair must be done before taking these models into comparisation process. When the complete model is obtained, first the foot mesh is cut into several sections towards the heel and toe. Then the area of each section(avail able contour) is calculated and compared with the area of equal sections in shoe last data base.

In summary, the problem of finding the best fitting shoe within a given shoe last data base, using input data taken from the 3D foot scan of client consists of two main sub problem:

- Filling hole in triangle mesh.
- Cutting the model to several sections to search though the shoe last data base for finding footwear fit.

Figure 1 shows process presented in this paper.

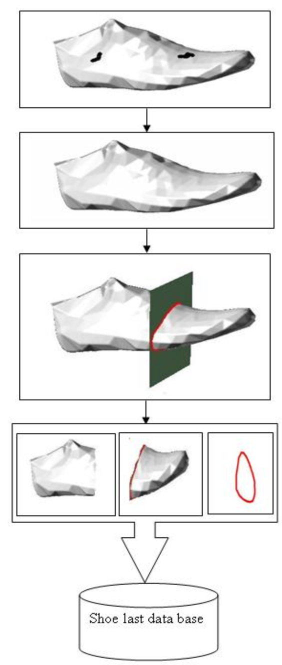

Figure 1: Steps for finding best fit from shoe last data base.

This paper is structured as follows: Section 2 reviews related work on footwear fit studies, while in Section 3 we present the filling hole in triangle mesh for building complete model. An efficient algorithm for cutting the 3D model is described in Section 4. Finally, conclusion and remarks are summarized in Section 5

2. RELATED WORK

Customer focus can influence today's business. Accurate fitting is very important factor in footwear manufacturing industry, for achieving commercial success. Mass customization starts with understanding individual customer's requirements and it finishes with fulfillment process of satisfying the target customers with

near mass production efficiency.
Properly constructed footwear improve compatibility between footwear and foot, thus contributing to fit and comfort [2], [3].
The issue of good shoe fit was posed as early as 1500 B.C. in 'Ebers Papyrus', which described a wide range of illnesses resulting from poor foot-shoe interaction [3].
Plastic tapes and Anthropometers are commonly used for obtaining measurements on people [4]. In traditional manufacturing, the device such as the Ritz Stick device [5], the Brannock device [6], the Scholl device [7], caliper and tape are always used for measurement of foot dimensions.
Depending upon the relative location of foot with shoe, the comparatively large dimensional differences can be causes either tight or loose shoe. Tight shoe will produce compression which may result in discomfort. On the other hand, when the shoe is loose, this may also lead to discomfort due to friction between the shoe and foot.
Properly constructed footwear may provide the right pressure and force at the different locations on the foot surface, and this may result in improved comfort, fit and foot health.
The design of shoe last, which represents approximate shape of human foot is the 'heart' of shoemaking because it mainly determines the foot shape, fit and comfort qualities.
Because of the complexity and the constraints imposed by the footwear manufacturing process, most importantly, the last manufacturing process, the custom footwear is expensive to produce.
In traditional, the process of foot measuring and making an accurate custom shoe last was always complicated and time consuming because the shoe maker must manually measure the specific consumer's foot and the last manufactured by last maker experience.
There are already some approaches in literature [8], [9], and [10]. The typical suggestion coming from literature is selecting a shoe last from a shoe last data base or deforming it into one that fits the scanned foot data. Authors in [11], quantify footwear fit and predict the fit-related comfort with colourcode mismatch between human foot and shoe last.
Li and Jneja [8], suggested to store front and back part of shoe last separately, to generate smooth surface between two given disjoined surfaces of the front and rear parts of the shoe last to obtain the new shoe last. This method is helpful for companies which already maintain library of last rear parts, so they need only front parts of last shapes to be designed as fashion suggests.
However, this method for custom tailored footwear designing is not very accurate because the consumers foot may change from time to time.
A meaningful way to evaluate footwear compatibility would be to determine the dimensional difference between the foot and shoe. The approach proposed in this paper is meant to cut the foot model to several section. Then the area of each section (available contour) is calculated and compared with the area of equal sections in shoe last data base. So that best fit can be obtained relatively automatically and quickly.

3. PRELIMINARIES

A *triangular mesh* is defined as a set of vertices and a set of oriented triangles that join these vertices. Two triangles are *adjacent* if they share a common edge.
A *boundary edge* is an edge adjacent to exactly one triangle. A *boundary vertex* is a vertex that is adjacent to a boundary edge. A *boundary triangle* is a triangle that own one or two boundary vertices. A *hole* is a closed cycle of boundary edges. A given hole is assumed to have no islands.
1-ring triangles of vertex are all triangles that share one common vertex. *1-ring edges* of vertex are all edges that share one common vertex and, all vertices on 1-ring edges of a vertex (except itself) are called *1-ring vertices* of the vertex.
A *vertex based topological* structure is used which records 1-rings vertices, 1-ring edges and 1-ring triangles of every vertex.
Average normal of the 1-ring triangular of the vertex is defined *vertex normal*. Figure 2 illustrated the preliminaries.

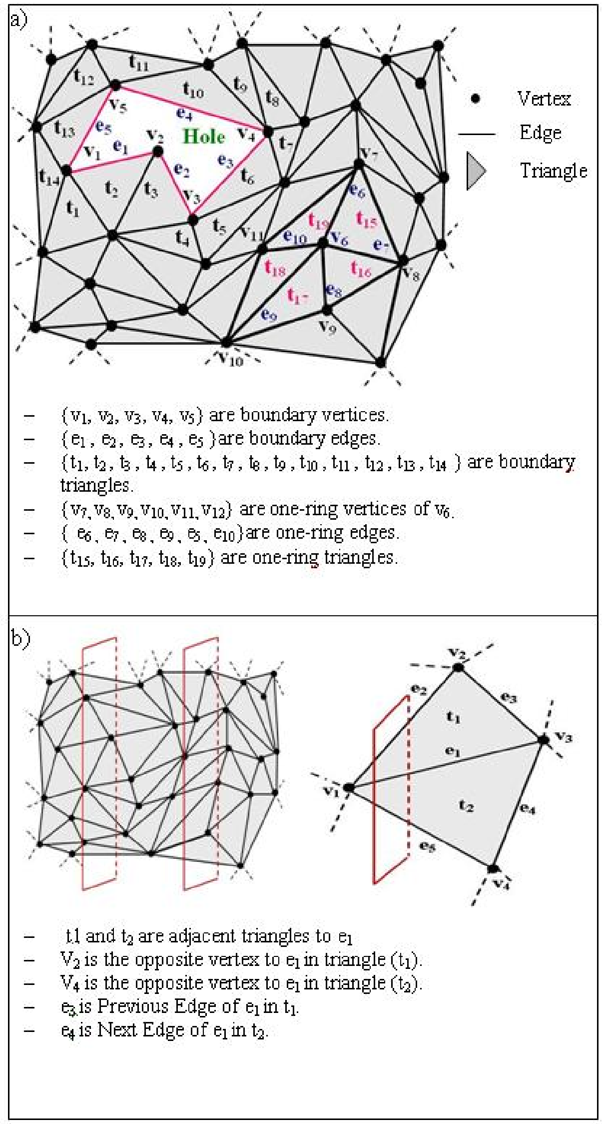

Figure 2: a) Preliminaries related to triangle mesh and hole. b) Preliminaries related to triangle mesh with intersection planes for cutting shape to several sections.

4. ACURATE 3D FOOT EXTRACTION

The development of acquisition devices that can easily and quickly acquire enormous number of surface points from a physical part have could possible automatic process of 3D models. However, several factors such as occlusion, low reflectance or even missing pieces in the original geometry can lead to incomplete data and this deficiency is not acceptable when the 3D model is taking into actual application. Thus certain repair must be done before taking these models into actual application.

In the literature we have surveyed, existing approach to fill holes in meshes can be distinguished two main categories: the geometric and non-geometric approaches.
Among the non-geometric approaches, authors in [12] detected the mesh areas that have to be filled with using volumetric representation. Davis et al [13] filled the gaps by applying volumetric process to extend a signed distance function through this volumetric representation until its zero set bridges whatever holes may be present. A similar approach has been developed by Authors in [14] for the simplification and the repairing of polygonal meshes. The advantage of this approach is working well for complex holes and drawback of current method include time-consuming and may generate incorrect topology in some case.
Considering the geometric approaches, the hole is filled in [15] with minimum area triangulation of its contour. Then the triangulation is refined so that the triangle density agrees with the density of the surrounding mesh triangles. Finally, the hole is smoothed with fairing technique based on an umbrella operator [16].
A satisfactory hole filling method should: 1. run in reasonable time. 2. be enable to patch an arbitrary holes for any model. 3. cover the missing geometry well. A hole filling process that is implemented here is quite similar to [17] and is summarized in following steps:

- Identify holes in triangle mesh. Holes can be identified automatically by looking close loop of boundary edges.
- Cover the holes with Advance Front Mesh technique.
- Modify the triangles in the initial patch mesh by estimating desirable normals instead relocating them directly.
- Rotate triangle by local rotation.
- Make algorithm more accurate by re-positioning these coordinate by solving the Poisson equation according to desirable normal and boundary vertices of the hole.
- Update the coordinate to make the smoothed patch mesh.

4.1 Hole patching

At first the hole is identified automatically by looking close loop of boundary edge. Then the Advance front mesh [18] technique is applied over the hole to generate an initial patch mesh as follows:

Step 1: The angle α_i between two adjacent boundary edges at each vertex v_i on the front is calculated.

Step 2: As Figure.3(a) depicts, when the angle α is less that or equal to $75°$, we simply connect the neighboring vertices of V_1, namely V_0 and V_2 , to form a new triangle. When the angle is larger than $75°$, but less than or equal to $135°$, as Figure.3(b) depicts, a new vertex V_3 is inserted along the bisection line of vectors V_0 V_1 and V_1 V_2 to determine the new vertex. When the angle is larger than $135°$, as Figure.3(c), two new vertices, V_3 and V_4 are inserted, equally distributed on two triple-section lines of vectors V_0 V_1 and V_1 V_2 .

Step 3: The distance between the new vertex and related boundary vertexes is calculated when the distance is less than given threshold, they should merge.

Step 4: Update the front and repeat the algorithm until the hole is patched with new triangles.

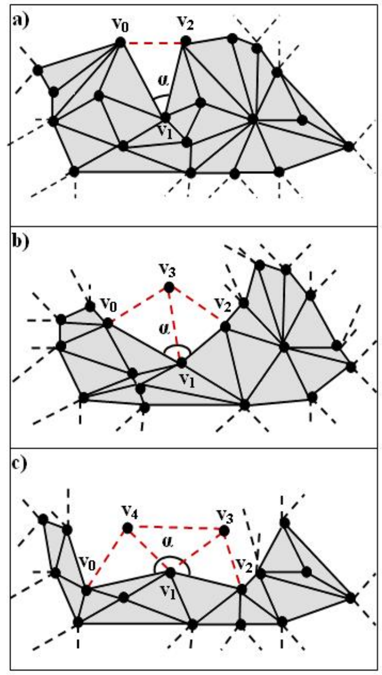

Figure 3: a) Rules for generating initial patch over the hole.

4.2 Harmonic-based desirable normal computing

The most important task of discrete harmonic function is to map a given disk-like surface $S_T \subset IR^3$ into the plane S^* which is introduced by Eck et al. [19] to computer graphic community. See Figure4.a.
Let S_T be in the form of a *triangular mesh* and V the set of vertices. If S_T has a boundary, then the boundary will be polygonal and we denote by V_B the set of vertices lying on the boundary and by V_I the set of interior vertices.
The goal is to find a suitable (polygonal) domain $S^* \subset IR^2$ and a suitable piecewise linear mapping $f : S_T \rightarrow S^*$ to minimize Dirichlet energy and it can determine by images $f(v) \in IR^2$ of vertices v. Overall, such mapping has two steps:

1- Find the boundary mapping, i.e.fix $f|_{\partial_{S_T}} = f_0$.

2- Find the piecewise linear mapping $f : S_T \longrightarrow S^*$ which minimizes the the Dirichlet energy for internal vertices.

$$E_D = \frac{1}{2}\int_{S_T} \|grad_{S_T} f\|^2 \tag{1}$$

Subjected to the Dirichlet boundary condition $f|_{\partial_{S_T}} = f_0$.
A quadratic minimization problem and reduction to solve a linear system of equations are the main advantages of current method. Consider one triangle $T = \{v_1, v_2, v_3\}$ in the surface S_T. Referring to Figure4.b, one can show that

$$\begin{aligned} 2\int_T \|grad_T f\|^2 &= \cot\theta_3 \|f(v_1) - f(v_2)\|^2 \\ &+ \cot\theta_2 \|f(v_1) - f(v_3)\|^2 \\ &+ \cot\theta_3 \|f(v_2) - f(v_3)\|^2 \end{aligned} \tag{2}$$

The normal equations for the minimization problem can be expressed as the following linear system of equations

$$\sum_{v_j \epsilon N_i} w_{i,j}(f(v_j) - f(v_i)) = 0 \qquad v_i \in V_I \tag{3}$$

Where

$$w_{i,j} = cot\alpha_{i,j} + cot\beta_{i,j} \tag{4}$$

and the angles are showed in Figure4.c. Here we have assumed the N_i denotes to 1-ring vertices of vertex v_i. The associated matrix is symmetric and positive and sparse, and so the linear system is uniquely solvable. The system can be solved efficiently with iterative methods such as conjugate gradient method. Note that system has to be solved three times. Once for x-, once for y- and once for z-coordinate. Now the desirable normal of all vertices in initial patch mesh is obtained. However, Poisson equation requires a discrete guidance field, i.e., w, defined on the triangles of the patch mesh. The guidance vector field is constructed by triangle rotation. Local rotation is applied to each triangle of initial patch mesh. Let n be the original normal of triangle and n' be the new normal of triangle that is calculated with desirable normal of vertices of triangle and c be the center of triangle. The rotation can obtained by rotating n to n' around c. See Figure5.

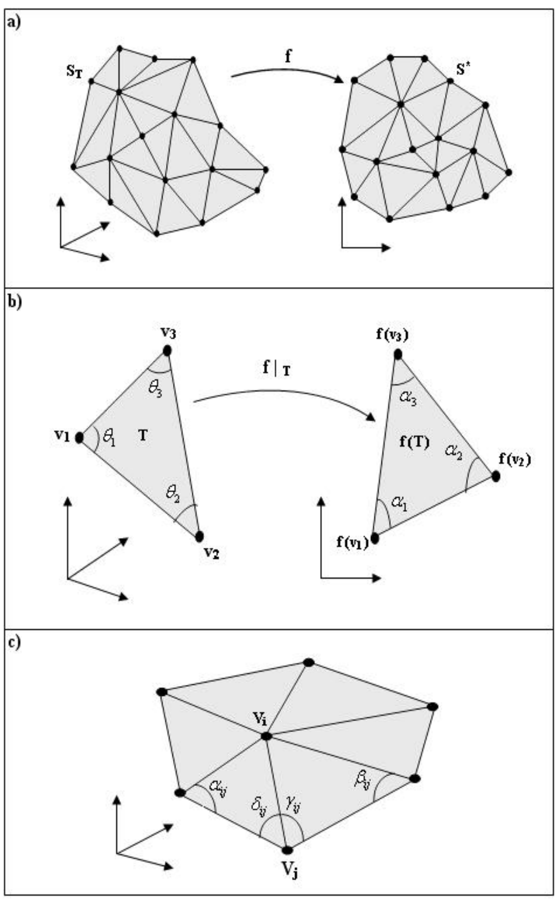

Figure 4: a) Piecewise linear mapping of a triangular mesh. b) Atomic map between a mesh triangle and the corresponding parameter triangle. c) 1-ring vertex of v_i and angles opposite to edge $v_i\ v_j$.

After rotation, the original patch mesh is torn a part and triangles are not connected any more and this torn triangles are used to construct a guidance vector filed for Poisson equation. Finally, the disconnected triangles are stitched by solving Poisson equation.

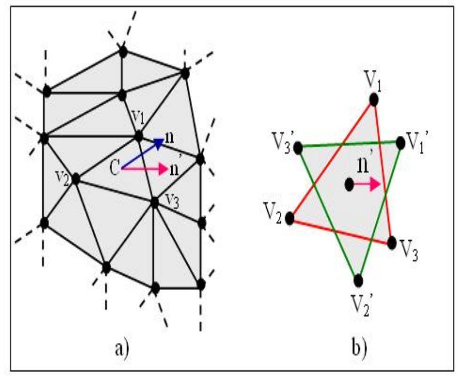

Figure 5: a) Initial patch mesh in triangle mesh, (b) A triangle red is initial patch mesh and its locally rotated is version green.

4.3 Poisson equation

In this section, we introduced the details of Poisson equation for reconstruction smooth and accurate patch mesh. We regard the mesh geometry (coordinates) as scalar function. The Poisson equation is originally appeared from [20]. The aim of this method is solving an unknown target mesh with known topology but unknown geometry (vertex coordinate). Poisson equation like Harmonic equation has to solved three time.

The Poisson equation with Dirichlet boundary condition [21], [22] is formulated as

$$\nabla^2 f = \nabla.w \quad over\ \Omega, \quad with\ f|_{\partial\Omega} = f^*|_{\partial\Omega} \tag{5}$$

where

- f is an unknown scalar function defined over interior of Ω.
- f^* is a known scalar function that provides the desirable values on the boundary $\partial\Omega$.
- $\nabla^2 = (\frac{\partial^2}{\partial x^2}, \frac{\partial^2}{\partial y^2}, \frac{\partial^2}{\partial z^2})$ is Laplacian operator.
- w is a Guidance Vector Filed and $\nabla.w = \frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} + \frac{\partial w_z}{\partial z}$ is the divergence of $w = (w_x, w_y, w_z)$.

Thus it can be defined as least-squares minimization problem:

$$\min_f \int\int_\Omega |\nabla f - w|^2 \quad with \quad f|_{\partial\Omega} = f^*|_{\partial\Omega} \tag{6}$$

A discrete vector field on a triangle mesh is defined to be a piecewise constant vector function whose domain is the set of point on the mesh surface. A constant vector is defined for each triangle, and this vector is coplanar with the triangle. For discrete vector field w on a mesh , its divergence at vertex vi can be defined to be

$$(div\ w)(v_i) = \sum_{T_k \epsilon N_{(i)}} \nabla B_{ik}.w|T_k| \tag{7}$$

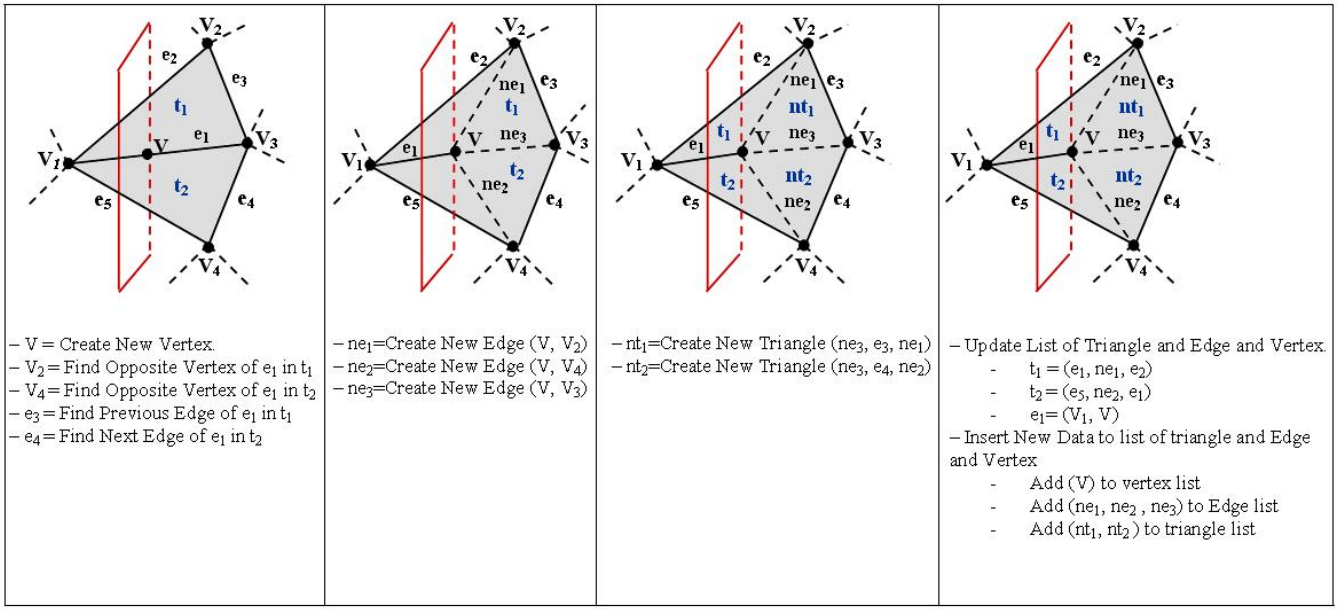

Figure 6: Steps for cutting shape to several sections

where N_i is the 1-ring vertices of v_i, $|T_k|$ is the area of triangle T_k, and ∇B_{ik} is the gradient vector of B_i within T_K.

The discrete gradient of the scalar function f on a discrete mesh is expressed as

$$\nabla f(v) = \sum_i f_i \nabla \phi_i(v) \tag{8}$$

with $\phi_i(.)$ begin the piecewise linear basis function valued 1 at vertex vi and 0 at all other vertices and f_i begins the value of f at v_i and it is one of the coordinate of v_i . The discrete Laplacian operator can determine as follow

$$\Delta f(v_i) = \frac{1}{2} \sum_{V_j \in N_i} (cot\alpha_{i,j} + cot\beta_{i,j})(f_i - fj) \tag{9}$$

where $\alpha_{i,j}$ and $\beta_{i,j}$ are the two angles opposite to edge in the two triangles sharing edge(v_i and v_j) and $N_i$is the set of the 1-ring vertexes of vertex v_i see Figure 4.c. Finally discrete Poisson equation is expressed as follows:$\nabla^2 f \equiv div(\bigtriangledown f) = \bigtriangledown w$
Discrete Poisson equation with Dirichlet boundary condition can be defined by the sparse linear system. It can be represented as the following form:

$$Ax = b \tag{10}$$

where the coefficients matrix A is determined by Eq.9 and the vector b is determined by Eq.7 and unknown vector x is the coordinate of all vertices on the patch mesh.
The smooth and accurate patch mesh is constructed as follow: *First*, Compute the gradient of each new vertex on the adjacent triangle by using Eq.8. *Next*, calculate the divergence of every boundary vertex by using Eq.7. *then*, determine the coefficient matrix A by Eq.9. vector b in this equation is determined by using divergence of all boundary vertices. *Finally*, solve the Poisson equation and obtain the new coordinate of all vertices of the patch mesh.

5. CUTTING 3D MODEL INTO SEVERAL SECTIONS

The input for the algorithm we developed is a triangle mesh file. The similarity search algorithm is based on the cutting foot triangle mesh into several sections towards the heel and the toe. Then the area of each section(available contour) is calculated and compared with the area of equal sections in shoe last data base.

5.1 Search for similarity estimation

The algorithm has the following steps summarized in Figure 6.

Step 1: Find the intersection of the cutting plane with the edge of a triangle and create new vertex.

Step 2: Choose the edge with the endpoints on the opposite sides of the intersection points.

Step 3: Find the next edge and the previous edge of the current edge.

Step 4: Build a new edge between the intersection point and the opposite vertices of the current edges triangles.

Step 5: Build a new triangle between the new edges.

Step 6: Update the triangles and vertices of the current edge.

Step 7: Add the new triangle and the edge to the list.

The output of our algorithm is set of vertices and edges related to each contour as illustrated in Figure 7. Let $V_c = \{v_1, ..., v_n\}$($v_i = (x_i, y_i, z_i) \in R_3$) be a set of "vertices" and $E_c = \{e_1, ..., e_n\}$ the set of "edges" associated to the contours.

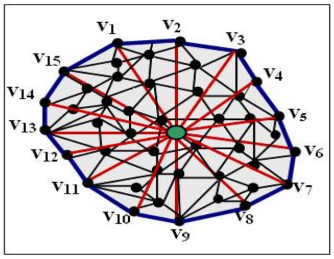

Figure 7: The illustrated contour after cutting mesh.

Step 1: : Let c_c be center of gravity of contour determined as follow:

$$c_c = \frac{\sum_{i=0}^{N} Vc_i}{N} \quad (c_c = (x_i, y_i, z_i) \in R_3) \qquad (11)$$

Step 2: For calculating the area of each contour. We divide the vertices of contours edge and center of gravity of contour in triangles. Let A_i be the area of each associated triangle in the contour and N be the number of triangles that associated with edges and center of gravity of contour. The area of each contour can be calculated as follow:

$$A = \sum_{i=0}^{N} A_i \qquad (12)$$

6. CONCLUSION

In this paper, a new approach for evaluating footwear fit within the shoe last data base is proposed. This new approach should clearly help to improve the users comfort and it could be the starting point for mass customization approach in footwear design.

Our method is based on an efficient algorithm to cut the model to several sections toward the heel and toe for extracting contours. Then the area of each contour is calculated and compared with equal sections in shoe last data base for finding best footwear fit. However, there is a need of post-processing step for filling hole in 3D models before taking these models in comparisation step. Thus, the advance front mesh technique is used to generate initial patch mesh. Then the desirable normal based on Harmonic function is calculated for modifying of initial patch mesh. Finally accurate and smooth triangle mesh are obtained by solving the Poisson equation according to desirable normals and the boundary vertices of the hole.

As a future work and development, the specific sections of the foot will be compared to corresponding sections of shoe latst's data base, so that new shoe lasts will be designed in such a way that they fit costumers feet completely.

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