A new modeling approach by global and local characterization

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Abstract

In this paper, we introduce a new approach for modeling closed surfaces. After working with several methods, we think that none really takes into account both the local and global characteristics of an object within a single coherent model. For instance, parametric surfaces endure changes of topology with difficulty and can mainly be used for local modifications of an object's surface. On the opposite, implicit surfaces allow a good global characterization of an object, thanks to a skeleton. However, the latter quickly shows its limits when it comes to small variations on an object's surface. After exposing several limits of classical existing approaches, we present the concept of a double characterization of an object, using two skeletons (intern and extern) and transition layers that link these two entities. The intern and extern skeletons define the object's topology, morphology and geometry in a different way than with the usual definition of a skeleton. In the end of this paper, we use this modeling approach in a reconstruction context.

Keywords: Shape characterization, geometric modeling, parametric surfaces, implicit surfaces, skeleton, triangulation, reconstruction

1 Introduction

In computer graphics, the concept of modeling is fundamental, and has to be dealt with before notions such as visualization, rendering, deformation or shape animation. In the case of simple shapes, modeling is not an obstacle. However we can think about the way we represent the shape, depending on the context we choose: pure modeling, reconstruction or conception. The underlying notion is shape description. It is all the more difficult to implement as the shape to model presents a high level of details.

In numerous cases, we come up against the problem of modeling. One type of approach allows to reconstruct one class of objects precisely, and another will model a great number of classes, but in a rough way. We then lose the global aspect of modeling, and the existing approaches show their limits: either we have a too global approach, or a too local one.

In an ideal context, we should be able to use a generic model, whose topology and morphology would be adaptable according to the problem we meet. Furthermore, we should have an exact geometric representation on a local plane, without perturbing the global structure. Immediate applications are automatic objects reconstruction and interactive conception.

In our context, we will limit ourselves to closed surfaces modeling, without any *a priori* on the topology (i.e. shapes said to be complex). Moreover, the only applicative domain that we will develop is reconstruction.

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This paper is divided in three parts. In the first part, we skim over different models' formalizations, and we try to extract the key characteristics of our approach, which we specify in the second part. We precise notions like *inner skeleton*, *external skeleton*, and *transition layers*. Then, we illustrate our modeling approach in a reconstruction context.

2 Different modeling formalizations

2.1 Parametric approach

2.1.1 Principle

Surfaces said to be defined by *control points* are defined by two parameters (u, v). They are manipulated according to a set of control points. These points are weighted and act on the local level as attractors of the surface. Amongst these surfaces, we can cite Bezier surfaces, B-splines and NURBS [6, 14]. Below we give an example of a Bezier surface formalization:

$$S(u,v) = \sum_{i=0}^{n_u} \sum_{j=0}^{n_v} P_{ij} B_{n_u,i}(u) B_{n_v,j}(v)$$

The P_{ij} points are the *control points*. They are the knots of a grid, called *control polyhedron*. The functions $B_{n_u,i}$ and $B_{n_v,j}$ are Bernstein polynomials which define weight functions.

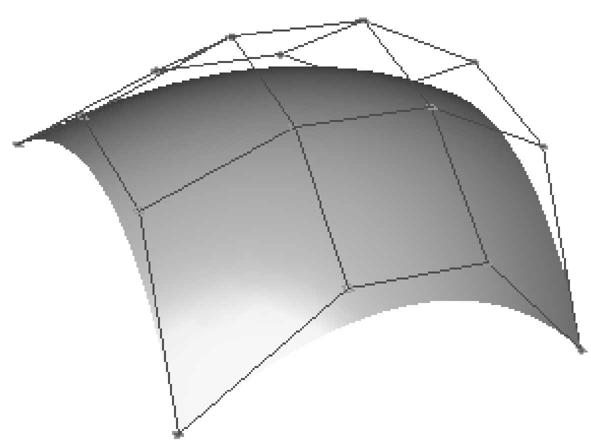

Figure 1: Control polyhedron and related Bezier patch.

2.1.2 Advantages

The control points allow us to have an intuitive and precise appreciation of the shape to model. It is particularly for this reason that such surfaces are frequently used in CADCAM. They allow an intrinsic local control on the object. The second main advantage is the ability to move easily on the surface, through the parameterization. Thus, the notions (always very local) of continuity, differentiation and curvature are exploitable in a coherent way.

2.1.3 Limits

Firstly, these representations need to set an *a priori* which is too important on the shape to model: a given parametric model conceived for surfaces homotopic to a sphere will generally be badly adapted to surfaces with a superior number of holes.

The second drawback with that kind of representation is that it's difficult to apprehend on a global scale. Indeed, if we wish to deform an initial model to a stretched one, we have to move a set of control points and to verify inducted transformations on the shape. In a word, it is not the vocation of parametric surfaces to set the global characterization of an object.

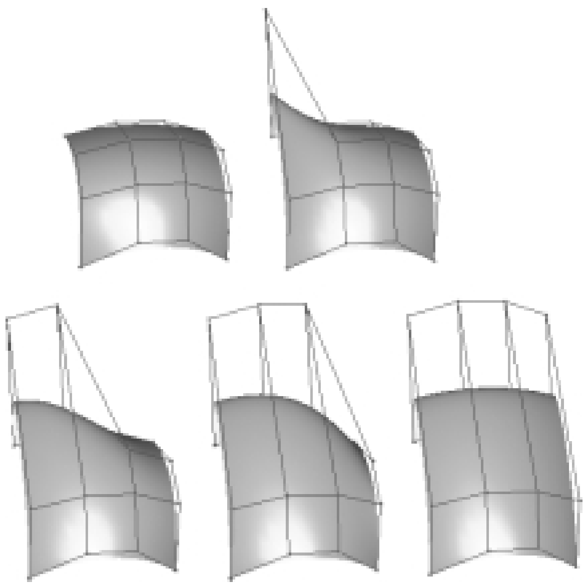

Figure 2: Moving control points on a Bezier patch.

Figure 2 illustrates the difficulty to use the control points to induce a global deformation: it is not their purpose.

2.2 Implicit approach

2.2.1 Principle

Behind the two words *implicit surfaces* a large field is hidden. In geometric modeling, we tend to use the notion of implicit surfaces defined by skeleton and potential function more often [13, 3, 16, 2, 12]. They are characterized by a set of geometrical primitives (the skeleton, in other words the *seeds*, which here are points, segments or triangles) and a potential function, depending on the distance between points and seeds. For a skeleton composed of γ_k seeds, the corresponding surface is the isosurface (The constant iso represents the isopotential for which the points are *on* the surface):

$$S_k = \{P | F_k \left(d(P, \gamma_k) \right) = iso\}$$

The implicit surface S is the isosurface defined by the sum (the *blending*) of the contributions of local potentials F_k applied to the N_γ seeds of the skeleton, with the aim of smoothing the junctions' zones and ensuring the geometric continuity of the surface.

The global potential function is:

$$F(P) = \sum_{k}^{N_\gamma} F_k(P)$$

The global surface related is the following isosurface, for the constant iso:

$$S = \{P | F(P) = iso\}$$

The skeleton can be a set of weighted points, or more generally a set of triangles, segments and points, or even all potential generating surfaces.

The potential function is a decreasing function, usually bounded, and that generally has one to three parameters which can be: the radius r of an isolated primitive, the influence radius R of a primitive and the slope k in a particular point. Below, a graphical example of potential function is given (figure 3); the exact formulation is not detailed because it is not the aim of our study [9, 11].

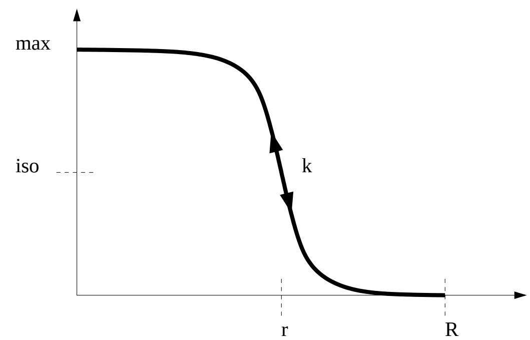

Figure 3: An example of potential function.

The main notion here is the skeleton, which defines the topological and morphological structure of the object. Figure 4 shows a skeleton composed by a triangle and by three segments, and the relative implicit surface.

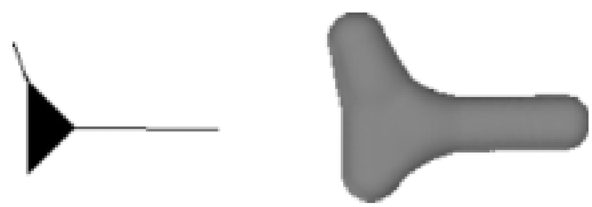

Figure 4: An implicit surface and its skeleton.

2.2.2 Advantages

Implicit surfaces with skeleton have two main advantages: first they allow a global control of the shape, thanks to the skeleton which is globally centered into the object. The second advantage is that we set no topological *a priori* on the shape to model, the structural information coming directly from the skeleton. This allows to model complex shapes on a topological plane. A simple modification of the skeleton is enough to generate a global deformation of the object (figure 5).

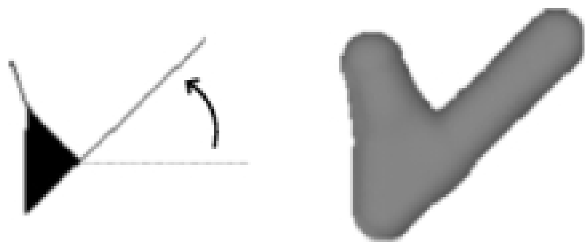

Figure 5: Global deformation of an object.

2.2.3 Limits

However, this approach has several drawbacks. First of all, it is impossible to move on such a surface. We can only know whether we are inside or outside of the volume generated by the surface, or on the surface itself. Then, calculus quickly becomes heavy when the number of primitives increases, making real time visualization difficult. Finally, although the kinds of modelizable shapes are infinite in theory, a too large number of primitives (to try to characterize details) becomes an obstacle when it comes to practical applications. These arguments point out to the same problem: it is difficult to have a local control with such surfaces because of the expanding number of seeds. Figure 6 shows a "well characterized" shape, and the same one with small variations which induce branches that are not characteristic of the global shape of the skeleton.

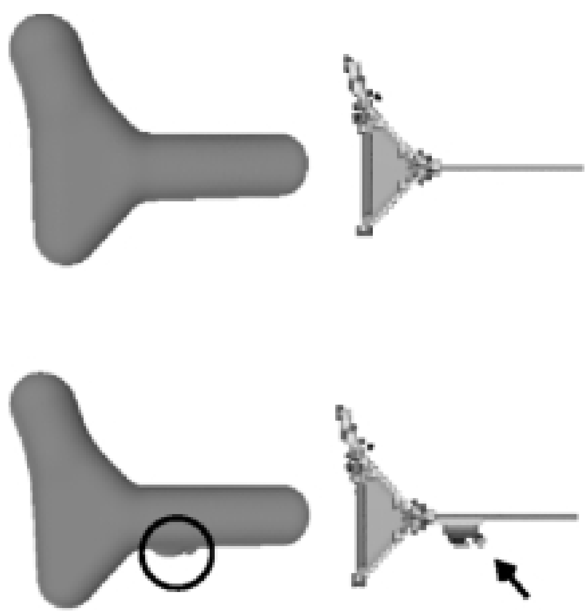

Figure 6: An example of skeleton perturbation.

2.3 Discussion

After studying two important movements of modeling, we can put forward the following relevant remarks:

- It should not be the role of the skeleton to take into account local variations on the surface, and yet its mode of computation and even its definition encourage it.
- It should not be the role of the control points to take into account global deformations depending on the structure. But the first modeling approach forces us to do so (through management attempts on an upper level of control point subsets).

We need a model which integrates these two approaches, or at least integrates their global and local characteristics.

Moreover, according to the complexity of the local variations, and to the size of the data, some approaches exist that permit to represent an object on many levels (multi-scale [8, 7, 15], subdivision surfaces [5, 10]). Thus, we would like a model which takes into account several levels of details. In a first time, this could be done by creating one articulation or more between the two functional levels of representation and their respective characterization (shape structure and surface details).

3 Definition of our model

3.1 Aims

Our discussion encouraged us to do the synthesis of several kinds of approaches, and we can then deduce the characteristics of our model. The most important according to us is the double local and global characterization of the shape. It has to be intrinsic for the model. It is the fundamental component which allows us to solve a lot of common problems for the modeling of complex shapes.

Considering we take into account articulations between the global structure of the shape and the surface details, one of our aims is to establish relevant displacements on the shape and on each characterization level.

3.2 Characteristics

Our model consists of several *layers*. If we consider that n layers are necessary for a *good* characterization of an object, we have:

- The first layer, the more internal one, that we call *inner skeleton* (layer 1). It defines the global structure of the shape, on topological and morphological planes.
- The external layer (the *external skeleton* or layer n) which characterizes local variations of the shape, regardless of the skeleton.
- *Transition levels* (layers 2 to $n-1$) which represent the articulations between the extreme layers.

The structure on each layer is a *complex* (a complex is any topological space that is constructed out of vertices, edges, and polygons by topological identification). However, in most cases, it is a triangulation, except for the inner skeleton which can be *non-manifold*.

The locating on each layer is done considering a set of parameters including components in barycentric coordinates (relatively to the nearest triangle of the area at stake) and an elevation value (signed scalar). This induces coherent movements depending on the layers and a geometric location between each layer.

For reasons of performance, according to the computer used, or to the size of the data, the shape representation with several levels of details is done by subdividing triangular primitives of the external skeleton.

3.3 Inner skeleton

This term belongs to implicit surfaces vocabulary, with reference to those defined by *skeleton* and potential function. It does not have the strict usual meaning which is the set of maximal balls' centers [4, 1], but remains an entity globally centered into the object.

The vocation of the skeleton is to define the global structure, the morphology and the topology (notion of *homotopic kernel*) of an object.

On the structural plane, it is a set of triangles, segments and points (it is the only non-manifold entity of the model, but it is really a *complex*). The edges define the connectedness relations between vertices. The triangles should not be confused, on a topological plane, with a set of three connected edges because the latter define a holed surface (change of homotopy).

A representation of the first surface approximation related to the inner skeleton is an implicit surface. The implicit volume so delimited has to be strictly included into the closed surface to model.

Figure 7 shows a 2D example of a shape and its inner skeleton.

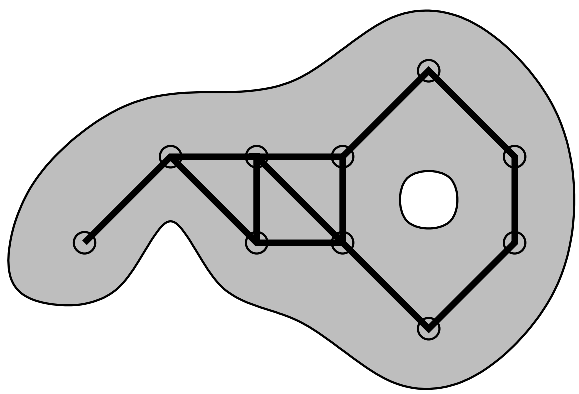

Figure 7: The inner skeleton of a shape.

3.4 External layer

The role of the external layer (or external skeleton) is to represent details and small variations, with a controlling ability, thanks to the vertices of the triangulation (notion of control points). The strong structural *a priori* being induced by the inner skeleton, morphological adjustments and the geometrical characterization are the vocation of this layer.

Thanks to the barycentric parametering (cf. §3.2), it is possible to move on the object in a coherent way. We can plate triangular primitives with a continuous surface (for example a parametric one), taking triangular Bezier patches or subdivision surfaces. The latter are linked to the multi-scale notion because they are as refinable as wanted. However, these considerations on the field of visualization go beyond the conceptual frame of our study.

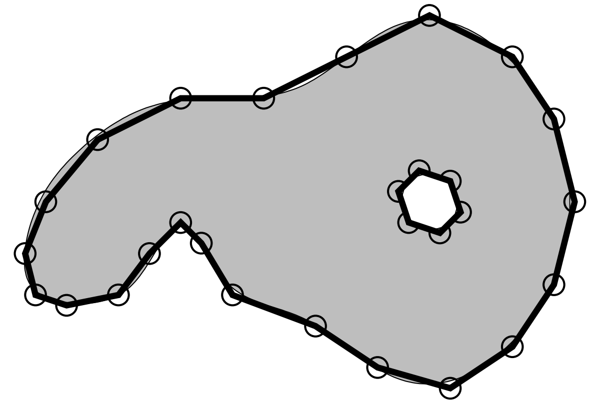

Figure 8: The external skeleton of a shape.

3.5 Transition layer

For a model composed of n layers, the transition layers are the layers 2 to $n-1$ (cf. §3.2). In practice, we can consider the use of one *intermediary layer*, and all in all the use of three layers for the model: inner, transition, external.

This layer represents the intermediate structure level, which makes the link between the global definition and the local characterization of an object. The inner and external representation levels are both as important and we want to characterize the articulation between them.

We define the transition layer as an intermediary triangulation between the two other skeletons. It induces a structure link allowing us to go from one layer to another (i.e. an element of the external layer can refer to an element of the inner layer and vice versa). The location is done thanks to the usual barycentric parameters, but the links are defined with a neighborhood list computed according to the *shortest distance* between a primitive of the transition layer and a primitive of another layer (see figure 9).

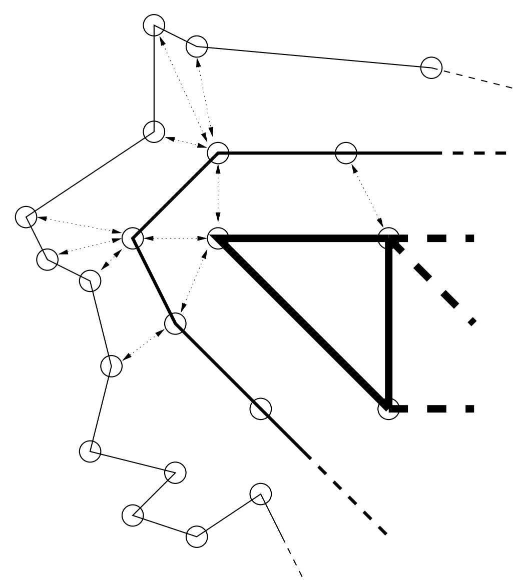

Figure 9: The three layers. The links between the primitives are obtained thanks to the use of the transition layer.

4 Using this modeling approach for a reconstruction context

After describing our model according to developed criterions in §3.1, we use it in this part within a reconstruction method. We will limit ourselves to this context without going to the field of conception (interactive management and manipulation of shapes) for practical reasons. Other specific developments to conception are being studied but are not yet finalized, particularly concerning ergonomic transformations (linked to the structure).

We apply our method of reconstruction to 3D objects represented for our study by digital volumes (i.e. sets of voxels included into a cubic grid).

The goal is not simply to characterize the boundary of the digital volume (a simple cloud of points) with the external layer (which is a triangulation independent from the inner skeleton, these two entities being linked by the transition layer). Indeed, we can ask ourselves if the external layer (in reconstruction) is sufficient to represent the related solid. It is, but two problems still need to be sorted out:

- We cannot easily find the inside points (whereas the model so defined allows it - as with implicit surfaces §2.2.3);

- We have no structural information on the shape, but this one is important even in reconstruction. For example, in the case of a model which evolves in time, it can be of interest to us (in terms of coding but also in terms of movement understanding) to implement this one on different levels.

4.1 Extraction of the inner skeleton

The inner skeleton of an object to reconstruct has to be a good structural, topological and morphological initialization. It must allow us to set a strong *a priori* on the intermediate layer. We obtain it by considering a sub-resolution of the object's digital volume. Let V_n be the starting volume, the optimal sub-resolution V_p is computed by embedding V_n into a cubic grid p^3 with $n = kp$, and considering the digital volume strictly included in V_n. k must be as big as possible but V_p has to be homotopic to V_n taking into account the fact that one voxel has 26 connected neighbors. The one voxel size of V_p is k^3 according to the voxels of V_n.

Figure 10 shows the digital volume of a *rubber ring* (a one-holed surface) into a 128^3 grid, its optimal sub-resolution (into a 8^3 grid), and also shows the voxels' centers connected with triangles or simple edges according to their connected neighborhoods.

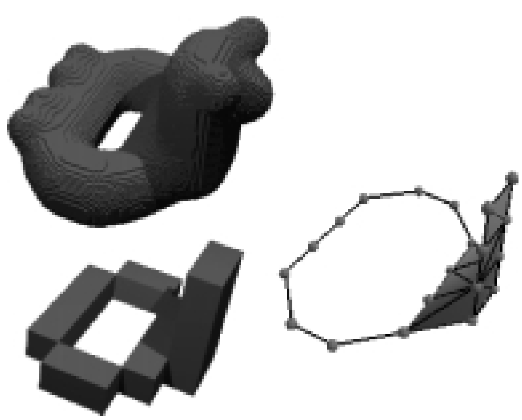

Figure 10: Digital volume, sub-resolution and inner skeleton.

4.2 External skeleton

The external skeleton represents the detail layer of the object to reconstruct. It is a triangulation where each vertex is a control point, for the future plating of the parametric surface for example. We can choose to select each point of the digital volume's boundary (the crust) for a maximal resolution. It is then possible, in order to reduce the complexity and to compress the structure, to simplify this triangulation [17].

Figure 11 shows the starting digital volume, its boundary (cloud of points) and the smoothed triangulation of its points.

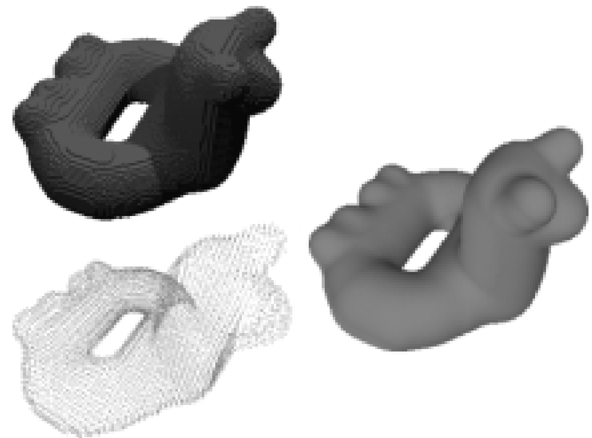

Figure 11: The external skeleton.

4.3 Transition layer

The intermediate structural level is computed by taking particular points on the implicit surface generated by the inner skeleton. This surface is defined by the vertices of the inner structure, considering a blending of blobs (cf. §2.2.1) with a radius of k. The points are obtained by taking intersections between the implicit surface and the rays from each seed. Since the volume V_p allowing to define the surface is strictly included into V_n, the control points of the transition layer are also included in V_n. We then triangulate these points to define the intermediary layer.

Figure 12 shows the implicit surface generated by the inner skeleton, particular points and the triangulated transition layer.

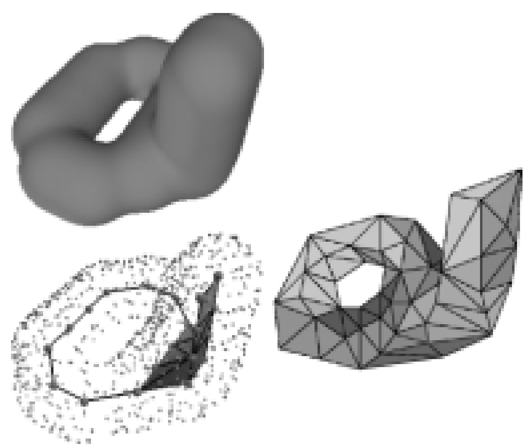

Figure 12: The transition layer.

Then we build a graph starting from the transition layer to match the triangles of the external layer (cf. §4.2) to the primitives of the inner skeleton. It represents a fundamental data structure for the locating according to each of the two levels. The links (the edges of the graph) are obtained by partitioning the space in Voronoï cells according to the vertices of the intermediate layer, and by linking the points of the external layer (or of the inner skeleton) according to their repartition into the cells.

5 Conclusion and future work

We have presented a modeling approach which aims to integrate the double characterization (local and global) of a 3D object. We defined the notion of intermediate layer to establish a relevant structural transition. Moreover, we have presented a reconstruction method based on this model. In addition to the surface characterization defined by the external layer, we have a structural information on the shape. It is also possible to locate on the surface and into the related volume.

We believe that this new approach will improve and will be applicable to very complex topologies, and to a large number of data points. This paper presents a modeling concept, and we still have to think about several points which we feel necessary to be dealt with as future work:

- Validation on large volumes of data, and particularly on the heart data into the frame of *Beating Heart*;
- Covering of the external layer with subdivision surfaces in order to have a multi-level refinement.

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